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A Simple Method of Stereoscopic Photomicrography

By Shin-ichi Shimadzu

(Received Dec. 15, 1936)

Abstract

A simple method of taking stereoscopic photomicrographs with an ordinary microscope is described.

Two optical axes of a binocular microscope are inclined differently towards the object, as shown in Fig. 1. If we give the object certain inclinations on both sides of the optical axis of an ordinary microscope as shown in Fig. 2, the relative positions of the objective lens and the object are the same as in Fig. 1. The writer took two photomicrographs by giving inclinations of about 10 degrees to the surface of the object on both sides of the optical axis of the microscope and the photomicrographs thus taken by looking at the object from two different directions were observed with a stereoscope.

Fig. 1

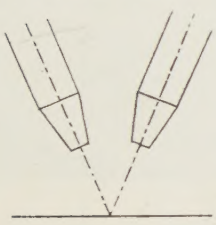
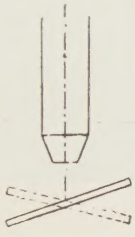


Fig. 2



The difference of the tilted angle of the surface of the object, which was about 20 degrees in the present experiment, corresponds to the optical angle in looking at the object with a binocular microscope; and the deepness of relief can be adjusted to any desired degree by merely changing the angle of tilting, irrespective of the amount of magnification.

When SUMP (Suzuki's Universal Micro-Printing) method[1] is employed, we obtain a good copy of the fine structure of the surface of a substance on the surface of a celluloid plate, and the photomicrographs of the replica can be taken with transmission light. The photographs reproduced in Plates I and II are obtained by means of the SUMP method. In these Plates Fig. 1 is the etched surface of a lead plate solidified from its melt, Fig. 2 the top portion of a rice-hulls,

1. By this method one obtains on a thin celluloid plate a detailed printing of the detailed surface structure of a solid body by pressing the plate, which has been previously smeared with amylacetate, on the surface of the solid body.

Fig. 3 a portion of a leaf of clover, and Fig. 4 the surface of a fibrous gypsum.

In these photographs the focusing is sharp at the middle portion, but not near the border. This is due to the tilting of the replica plate and is unavoidable with high magnification. Limited by this fact, the usual magnification employed in the present experiment was less than 200. However, with narrower visual field the magnification can be still more increased; and the writer could take some stereoscopic photomicrographs magnified more than 600 times.

In conclusion the writer wishes to express his sincere thanks to Prof. U. Yoshida for his kind guidance during the course of this experiment.

Plate I

Fig. 1

Etched surface of a lead plate (×130)

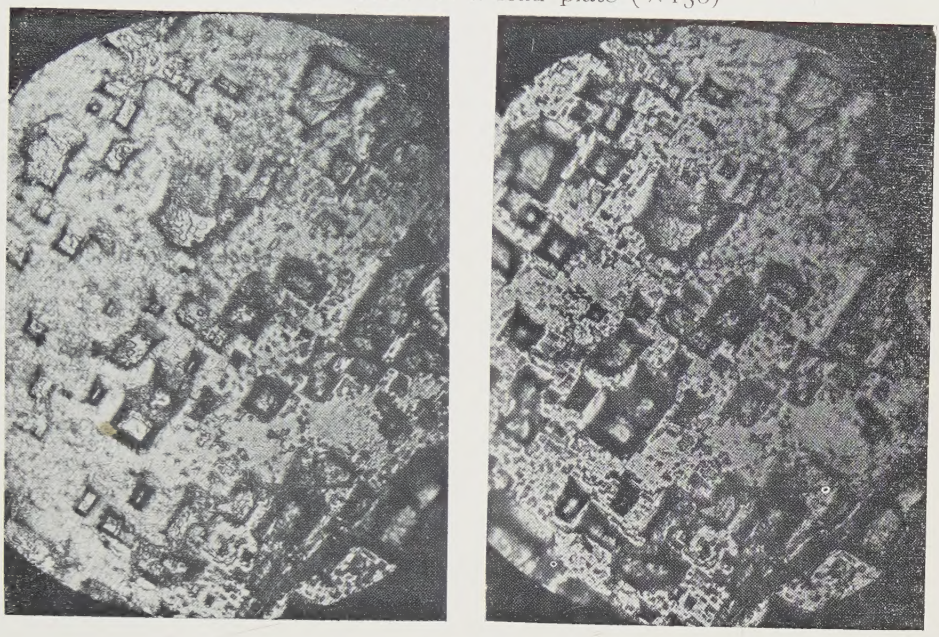


Fig. 2

Rice-hulls (×20)

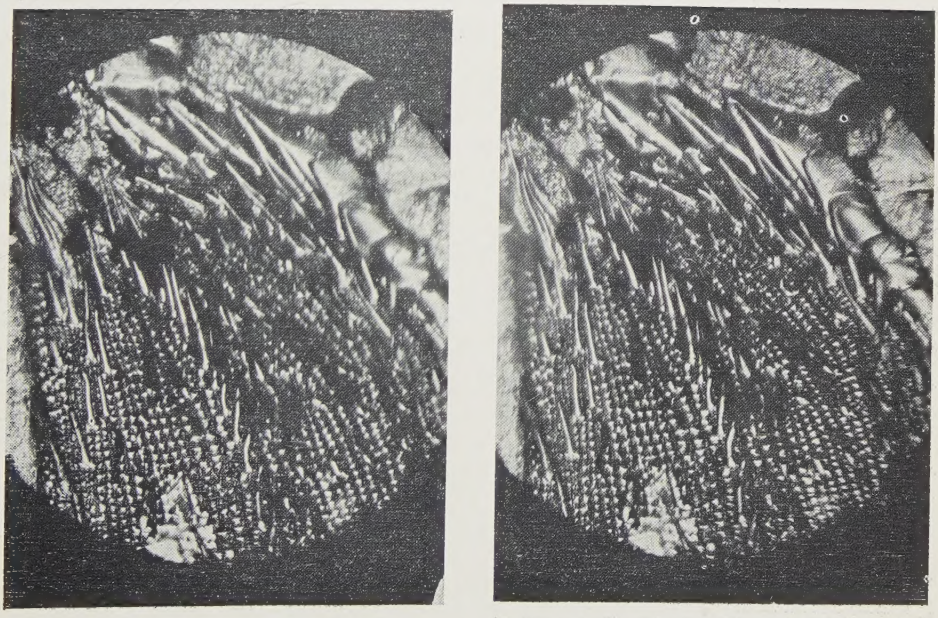


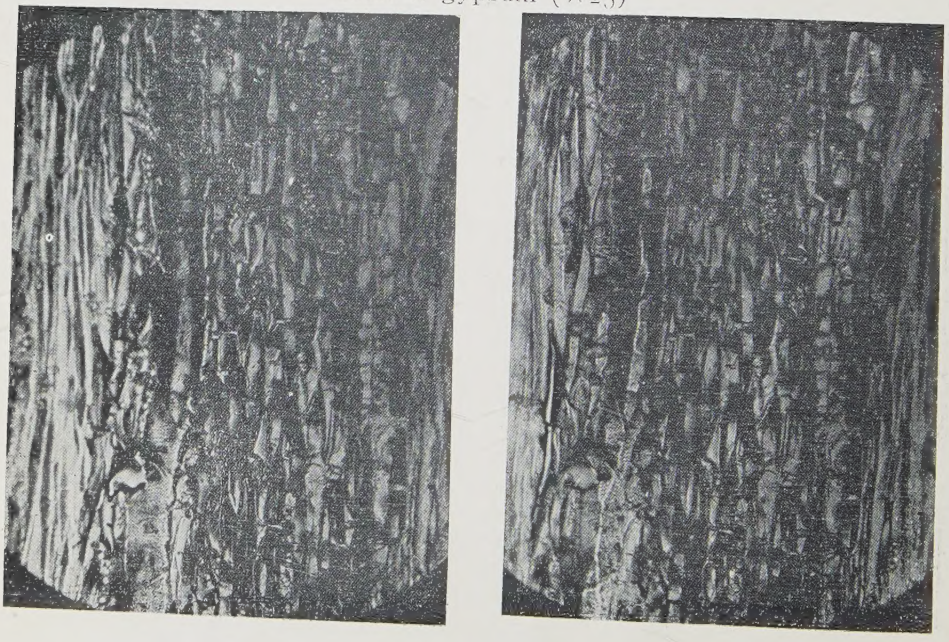
Plate II

Fig. 3

Leaf of clover (×20)

Fig. 4

Fibrous gypsum (×25)



Proper Oscillations of Lake-shelves

By

Takaharu Nomitsu, Katakusu Habu and Mitsutoshi Nakamiya

(Received Jan. 12, 1937)

I. Introduction

We have previously shown[1] that the long-period surface oscillations (of one to two hours) on the open coasts, which have hitherto been unexplainable as seiches of neighbouring bays, must be attributed to the proper oscillations of the continental-shelf-sea. We proved also theoretically[2] that the oscillations of the shelf-sea can be dealt with in a manner similar to the way in which the oscillations of the bay are treated. We may expect that in the lake also, at the portion where the shelf is fully developed, oscillations of the same kind will occur. Actually, oscillations of about 5 min. period (Fig. 4) recorded during the Muroto typhoon by the limnogram installed on the Imazu coast of Lake Biwa, can not been explained by any usual oscillating area forming a bay. One of the writers (Nomitsu) has suggested in another paper[3] that they are oscillations of the neighbouring lake-shelf. According to this suggestion, the present work has been done. We describe herein the results of our own limnological observations and minute soundings over the well-developed shelves at Imazu and Ômizo in Lake Biwa and examine whether the observed periods coincide with those calculated for the corresponding lake-shelves.

II. Sounding

When we look at the depth map[4] of Lake Biwa drawn by the Kôbe Marine Observatory, we find that shelves develope in the north-western portion of the lake, most remarkably in the neighbourhood

1. T. Nomitsu and K. Habu: Proper Oscillations of the Sea of Continental Shelf. These Memoirs, A, **18**, 247 (1935).

2. T. Nomitsu: A Theory of Tunamis and Seiches produced by Wind and Barometric Gradient. These Memoirs, A, **18**, 213 (1935).

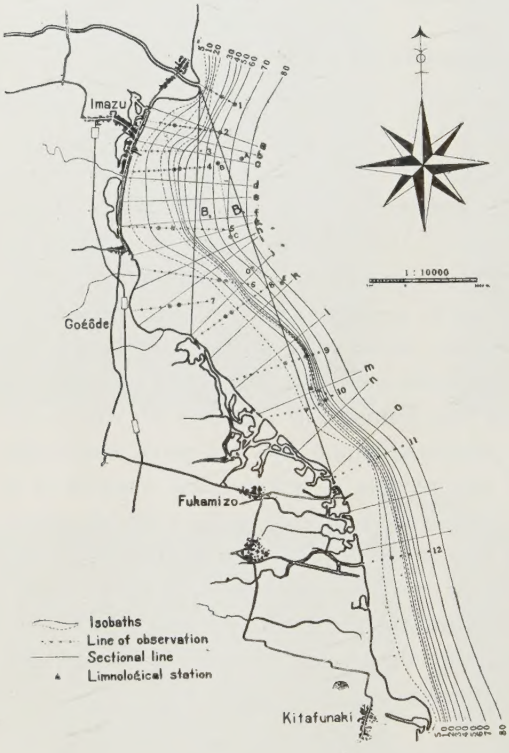
3. T. Nomitsu: Surface Fluctuations of Lake Biwa caused by the Muroto Typhoon. These Memoirs, A, **18**, 227 (1935).

4. 海洋氣象臺彙報 **8**, (1926). Its reproduction is given in these memoirs, A, **18**, 223 (1935).

of Imazu and Ômizo and next in the vicinity of Kaizu. But since, in this map, there are almost no soundings in the lake-shelf, and the observing points about the shelf margin are very few, we had to make for ourselves soundings sufficiently minute for our present purpose.

The soundings in the neighbourhood of Imazu are shown in Fig. 1.

Fig. 1



We established 12 sounding lines from the shore, and one line perpendicularly to them in the off-shore. Whenever possible we chose for the origin of the observation lines such points as the end of piers or the mouth of a river, points which are definitely indicated on the map issued by the Land Survey Department and can be easily found from the off-shore. Where no such landmarks were available, we indicated the starting points by flag-staffs. For the depth measurement, a sounding rope with lead and a modified Lucas machine were prepared, and for the determination of the observing station we prepared a sextant, a simple telemeter and a 100 m. measuring rope (to which were attached many small floats in order that it could be stretched over the water in a line as nearly straight as possible), and many rubber toy-balloons.

To secure our soundings, we used two Japanese junks which we rowed in a straight course by fixing our eyes on a definite object such as a point of a mountain or an island situated far on the other shore. When we had rowed about 100 m. (a few times only 70 m.), we stopped the boats, dropped the anchor, floated one of the rubber toy-balloons with lead sinker as a bouy, and measured the distance between

the balloons with the measuring rope, and the distance from the shore by the simple telemeter for reference. Further, using the sextant we determined the positions of a few points (marked ⊙ in Fig. 1) on each observation line, the direction of which thus were known by these points and the points of origin.

The depth at each station was measured by both the Lucas machine and the sounding rope, taking the mean of the two as the real depth there. But as there were no great differences between them, sometimes we used only the sounding rope, mainly to save our labor and time. The results are tabulated in Table 1.

Table 1

Result of soundings near Imazu.

I-line

Station No.	Depth (m)	Distance from the shore (m)
1	10.5	50
2	23.5	100
3	28.7	200
⊙ 4	30.7	300
5	55.2	400
⊙ 6	62.4	500

II-line

Station No.	Depth (m)	Distance from the shore (m)
1	5.5	180
2	8.4	250
3	11.1	320
⊙ 4	12.2	390
5	13.4	460
6	15.1	530
7	17.1	600
8	20.6	670
9	21.8	740
10	33.0	810
11	49.7	880
12	57.2	950
⊙ 13	60.5	1020

III-line

Station No.	Depth (m)	Distance from the shore (m)
1	8.6	148
2	10.5	240
3	15.1	370
4	17.9	463
5	27.4	
6	29.8	685
7	55.6	926

IV-line

Station No.	Depth (m)	Distance from the shore (m)
1	6.8	70
2	8.1	140
3	9.2	210
4	10.8	280
5	12.4	350
6	14.6	420
7	16.9	490
8	20.4	560
⊙ 9	25.6	630
⊙ 10	32.4	700
11	39.5	770
12	43.6	840
13	46.6	910
14	48.7	980
15	50.5	1050

V-line

Station No.	Depth (m)	Distance from the shore (m)
1	4.7	100
2	6.2	200
3	8.4	300
4	10.4	400
⊙ 5	11.4	500
6	12.5	600
⊙ 7	14.2	700
8	16.0	800
9	32.0	900
10	47.6	1000
11	49.3	1100
12	57.3	1200
13	59.1	1300
14	63.0	

VI-line

Station No.	Depth (m)	Distance from the shore (m)
1	2.1	200
2	2.0	300
3	2.0	400
4	2.0	500
5	2.0	600
6	2.8	700
7	2.8	800
8	3.8	900
9	6.5	1000
⊙ 10	9.3	1100
⊙ 11	13.4	1200
12	15.0	1300
13	41.3	1400
14	53.3	1500

VII-line

Station No.	Depth (m)	Distance from the shore (m)
1	2.4	130
2	2.4	260
3	2.6	390
⊙ 4	2.6	520
⊙ 5	2.9	650
6	3.2	780
7	2.9	910
8	2.5	1040

VIII-line

Station No.	Depth (m)	Distance from the shore (m)
1	2.1	0
2	2.5	100
3	2.9	200
4	2.3	300
⊙ 5	2.2	400
⊙ 6	3.0	500
7	3.8	600
8	5.3	700
9	6.2	800
10	22.0	900
11	43.0	1000
12	54.5	1100
13	60.3	1200

IX-line

Station No.	Depth (m)	Distance from the shore (m)
1	1.5	130
2	2.5	260
3	2.9	390
4	2.9	520
5	3.4	650
6	3.9	780
7	4.6	910
⊙ 8	5.3	1040
9	6.0	1170
10	7.1	1300
⊙ 11	13.5	1430
12	59.1	1560
13	66.0	1690

X-line

Station No.	Depth (m)	Distance from the shore (m)
1	1.8	100
2	2.1	200
3	2.8	300
4	3.5	400
5	3.5	500
⊙ 6	5.5	600
7	8.2	700
8	13.0	800
⊙ 9	42.4	900
10	65.0	1000
11	51.3	1100
12	32.0	1200
⊙ 13	13.5	1300

XI-line			XII-line		
Station No.	Depth (m)	Distance from the shore (m)	Station No.	Depth (m)	Distance from the shore (m)
1	0.65	37.6	1	1.1	0
2	1.9	167.6	2	2.0	130
3	3.2	297.6	3	5.0	260
4	3.6	427.6	⊙ 4	10.3	390
5	3.9	557.6	5	13.5	520
6	32.0	687.6	6	36.9	650
7	57.8	817.6	⊙ 7	60.0	780
8	68.2	947.6	8	68.4	910
9	75.0	1077.6			

Next, *the soundings in the neighbourhood of Ômizo* were taken along 14 observation lines, as shown in Fig. 2. First we observed the mouth line of the bay and set 4 bouys of wooden frame with lead sinker, intending to make convenient further observations in the bay.

Fig. 2

This time, the position of each observation point was decided by "the two theodolites method," the theodolites being set on the ■-marks in Fig. 2, and the depth was measured only with the sounding rope. The irregularity of arrangement of the observation points is due to the fact that the two theodolites method made us so easy as we were not so precise in fixing the ship's course, and worked on no anchoring, the ship thus drifting by the lake current. The results are given in Table 2.

As for *the neighbourhood of Kaizu*, we did not sound ourselves, because there are many points sounded by the Kôbe Marine Observatory, up to fairly near the shore.

Table 2

Result of soundings near Ômizo. (Angles are measured from the base line connecting the two theodolites.)

I–line

Station No.	Depth (m)	Angle read at H-station	Angle read at E′-station
1	8.8		
2	13.9		
3	16.2		
4	20.7		75° 01′
5	21.8		74 20
6	49.4		76 51
7	72.1		77 18

II–line

Station No.	Depth (m)	Angle read at H-station	Angle read at E′-station
1	1.0		50° 49′
2	2.6		54 05
3	3.4	253° 19′	59 04
4	5.5	243 08	64 19
5	12.1	216 54	78 30
6	18.9	202 39	81 19
7	20.0	192 53	79 09
8	33.1	200 05	83 49
9	38.6	190 33	87 30
10	52.0	175 50	88 46
11	51.7	173 05	69 40
12	62.0	167 35	75 10

III–line

Station No.	Depth (m)	Angle read at H-station	Angle read at E′-station
1	2.5		46° 52′
2	6.0		55 09
3	10.0		67 21
⊙ 4	10.5	230° 48′	81 22
5	23.8	215 09	
6	23.6	211 19	
7	38.0	227 41	
8	43.2	204 53	
9	48.2	200 54	

IV–line

Station No.	Depth (m)	Angle read at A-station	Angle read at E-station
1	8.0	39° 31′	127° 09′
2	12.0	42 46	142 40
3	13.0	46 15	147 20
4	13.6	51 02	154 55
5	24.4	59 44	154 31
6	47.0	73 01	186 23
7	54.0	75 11	184 30
8	27.7	62 51	177 47
9	27.7	55 09	201 27

V–line

Station No.	Depth (m)	Angle read at A–station	Angle read at E-station
1	3.1	21° 04′	280° 37′
2	6.0	23 41	271 44
3	8.0	26 48	261 09
4	9.7	30 55	248 27
⊙ 5	11.0	(at C.) 111 04	232 12
6	12.0	40 03	222 24
7	12.7	43 44	216 29
8	13.0	47 57	210 05
9	15.0	50 40	206 32

VI–line

Station No.	Depth (m)
1	3.7
2	4.5
3	6.0
4	8.0
5	8.5
6	9.2
7	10.5
8	8.5

VII–line

Station No.	Depth (m)	Angle read at C-station	Angle read at E-station
1	11.2	94° 24′	257° 05′
2	11.5	97 29	232 20
3	11.2	106 13	228 06
4	11.5		
⊙ 5	11.0	121 52	235 31
6	10.5		
7	10.0		
8	8.8		
9	5.0		

VIII–line

Station No.	Depth (m)	Angle read at C-station	Angle read at E-station
1	10.0		
2	10.2		
3	10.0		
4	10.1	154° 34′	228° 40′
5	10.0	165 49	228 21
6	9.1		
7	8.8		
8	8.7		
9	6.2		
10	2.4		
11	3.7		
12	4.9		

IX–line

Station No.	Depth (m)	Angle read at A-station	Angle read at E-station
1	2.8		
2	4.0	5° 12′	259° 52′
3	9.0	16 08	
4	10.2		
5	10.0		243 51
6	10.5		235 13
⊙ 7	11.1	139 21	228 13
8	11.5	(at C.)	219 31
9	15.5		207 50
10	22.5	59 10	198 40
11	38.0	63 51	194 20
12	39.0	68 52	190 21
13	42.5	71 46	182 16
14	60.0	82 36	174 06

X–line

Station No.	Depth (m)	Angle read at A-station	Angle read at E-station
1	3.5		
2	6.0		
3	8.5		
4	9.0		
5	10.0		
⊙ 6	10.0	(at C.)	
7	10.0	143° 51′	228° 04′
8	12.0	38 00	
9	21.8	44 17	
10	40.7	56 29	
11	50.5	57 30	

XI–line

Station No.	Depth (m)	Angle read at A-station	Angle read at E-station
1	10.6		
2	11.0		220° 07′
3	11.4	66° 02′	216 49
4	13.8	73 30	213 12
5	30.5	76 23	211 19
6	35.8	78 06	210 12
7	36.6	79 48	209 02
8	43.3	85 04	205 33
9	45.0	86 13	
10	46.3	86 52	203 37
11	51.9	91 21	200 18
12	53.6	94 01	196 53
13	55.1	95 15	195 30
14	55.1	98 14	194 55

XII–line

Station No.	Depth (m)	Angle read at C-station	Angle read at E-station
1	2.1		
2	4.1		
3	3.7		
4	6.7		
5	7.5		
6	8.9		
7	9.6		
8	10.4		
9	10.8		
10	10.9		
⊙ 11	12.0	173° 01′	223° 19′
12	11.2		
13	11.8	182 32	

XIII-line

Station No.	Depth (m)	Angle read at E-station
1	12.6	224° 21′
2	24.2	220 32
3	29.7	219 02
4	37.6	215 19
5	43.5	211 39
6	54.6	213 15
7	55.0	211 37
8	55.4	210 46
9	55.9	209 47
10	58.0	207 54
11	60.0	206 20
12	60.6	205 04

XIV-line

Station No.	Depth (m)	Angle read at A-station	Angle read at E-station
1	2.3		
2	23.6	206° 55′	
3	34.1	196 41	224° 20′
4	38 4	191 57	222 06
5	44.1	187 40	
6	50.9	182 11	
7	53.9	179 32	218 39
8	56.0	175 34	217 08
9	59.4	167 50	214 36
10	60.1	163 54	212 58
11	61.8	161 29	211 50
12	60.6	154 42	210 34
13	61.0	152 06	209 55
14	60.4		

III. The records of the seiches

This autumn, at Imazu, Ômizo, Kaizu and other places we established non-reducing limnimeters of the bouy-type made in our own workshop, and made many records of the seiches. We show only some examples in Figs. 3–4. We reproduce also the Imazu record of the Muroto typhoon obtained by the Section of Public Works of Siga Prefecture, for the record is an apt illustration for the present purpose. In these records special attention must be paid to the undulations of the 5 min. period at Imazu and the 15 min. period at Ômizo. That these two are certainly the oscillations of the lake shelf is proved in the following articles, and can not be explained by any other oscillating area. Beside these, several other periods can be found in our own records, the aforementioned Bulletin of the Kôbe Marine Observatory and the report of Dr. S. Nakamura and Dr. K. Honda.[1] At Imazu the seiches of period of about 9 min. and 30 min. occur

Table 3.

Observed periods (Thin type means weak rare occurrence)

Location	Observed period (in min)				
Imazu	**5**	**9**	**30**	22	71
Ômizo	**15**	**8–9**	**12**	17–18	
Kaizu	9	11–12	**30**		

1. S. Nakamura and K. Honda: Seiches in some Lakes of Japan. Jour. Coll. Sci. Imp. Univ. Tokyo, Vol. **28**, Art. 5 (1911).

Fig. 3

Limnograms at Ômizo (Yanagawa, Dezaike)

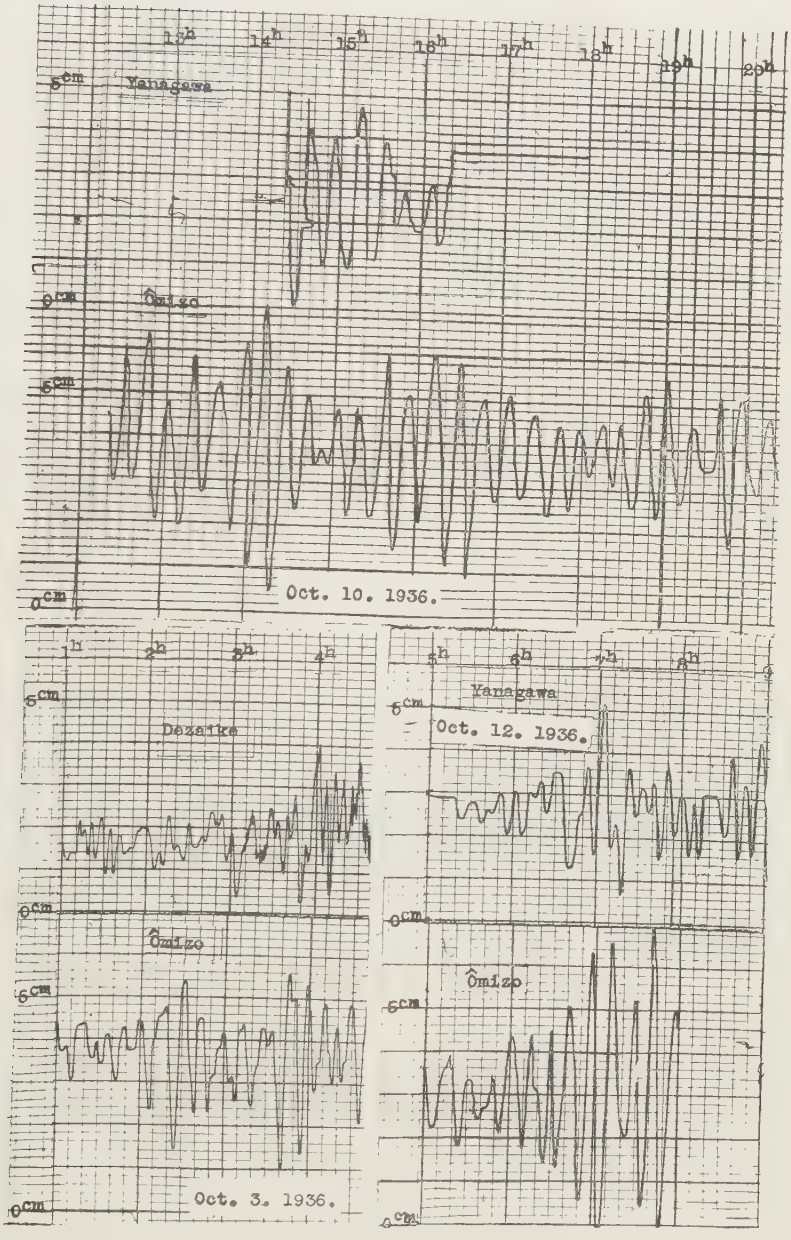
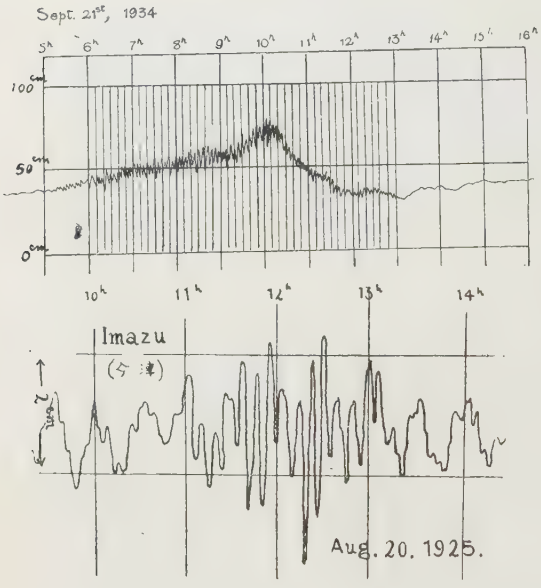
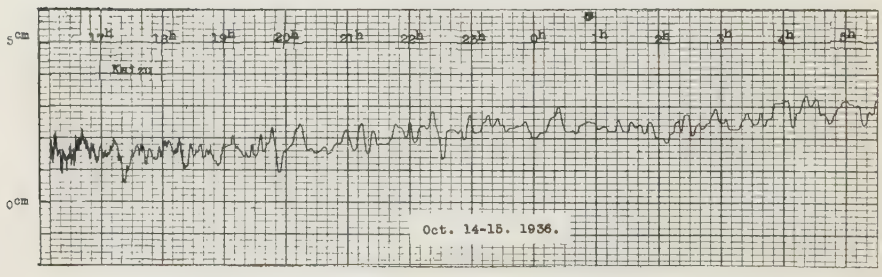


Fig. 4
Limnograms at Imazu and Kaizu.

(A) Imazu



(B) Kaizu



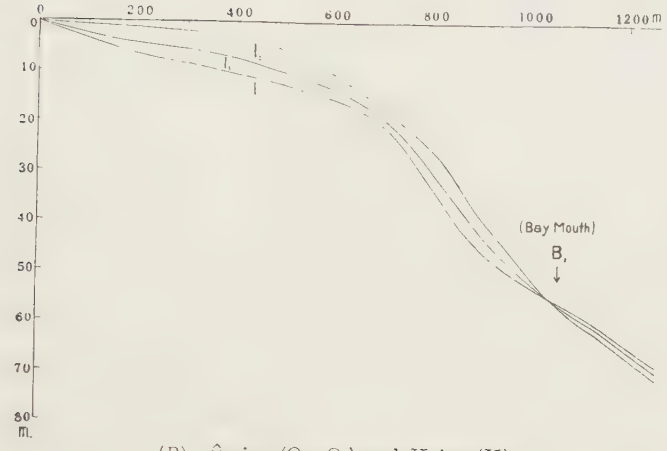
frequently and slight oscillations of 22 min. and 71 min. occur rarely. At Ômizo the strong seiches of 8–9 min. and 12 min. often appear and sometimes about 18 min. in slight intensity. At Kaizu about 30 min. is the habitual period and 9–12 min. seiches are not rare. Table 3 gives the summary of the above described.

IV. The lake-shelves and other oscillating areas

If we draw cross sections of the lake bottom according to the data given in Chap. 1, we see the distinct formation of the lake shelf (Fig. 5 & Table 4). Especially at Ômizo the shelf is much more conspicuous, and at Imazu it is a little narrower and more gentle, than we would expect judging from the map prepared by the Kôbe Marine Observatory.

First, *for the neighbourhood of Imazu*, I_1 gives the mean of the nine cross sections corresponding to lines a to i which are drawn orthogonally to the contour lines (cf. Fig. 1). Similarly I_2 is the mean of the cross sections from j-line to o-line, and I_3 represents the mean

Fig. 5
Cross sections of lake bottom.
(A) Imazu.



(B) Ômizo (O_1, O_2) and Kaizu (K).

of all cross sections from a-line to o-line. The lake shelf against the inlet from Imazu to Gogôde is I_1, and the lake shelf south of it is I_2, while I_3 is the mean shelf covering the whole region whose center is situated at Gogôde.

Now let T_1, T_2, T_3 denote the corresponding periods of the proper oscillations of each

Table 4
Mean cross section of the lake bottom.

Distance from the shore	I_1 Depth	I_2 Depth	I_3 Depth	Distance from the shore	O_1 Depth	O_2 Depth
62.5 m.	2.5 m.	0.4 m.	1.7 m.	200 m.	3.5 m.	3.1 m.
125.0	4.6	0.8	3.1	400	6.9	6.3
187.5	6.3	1.2	4.2	600	8.2	9.1
250.0	7.5	1.6	5.1	800	9.6	12.8
312.5	8.9	2.0	6.1	1000	10.3	16.4
375.0	10.0	2.5	7.1	1200	11.2	24.2
437.5	11.3	3.6	8.2	1400	11.5	36.0
500.0	12.6	5.4	10.0	1600	12.3	45.0
562.5	14.4	8.7	12.1	1800	14.6	54.2
625.0	16.8	12.2	14.8	2000	16.4	58.0
687.5	20.1	16.4	18.6	2200	28.3	
750.0	26.3	21.5	24.4	2400	37.8	
812.5	35.7	27.1	32.2	2600	43.1	
875.0	43.8	36.1	40.7	2800	50.3	
937.5	49.0	43.5	46.7	3000	54.4	
1000.0	53.1	51.8	52.8	3200	59.4	
1125.0	60.3	62.2	61.1			
1250.0	68.8	71.7	70.0			

shelf I_1, I_2, I_3 respectively, and calculate them by the formula

$$T=\frac{4L}{\sqrt{g\bar{h}}}=4\int\frac{dL}{\sqrt{gh}},$$

L: The breadth
h: The depth,

then we get the following table. The upper values in the table are obtained by using the usual mean (arithmetical) as the mean depth $\bar{h}$, and the lower by using the integral mean of $dL/\sqrt{gh}$ for every 100 m. of dL. In any case, it is evident that the lake shelf about Imazu (I_1 or I_3) has a period of about 5 min.

Table 5

Calculated periods for the shelf near Imazu.

T_1	T_2	T_3
4.5 min.	7.5 min.	4.9 min.
5.8	8.7	5.6

Besides the shelf, what area including Imazu may generate the seiches shown in Table 3? There is the main body of Lake Biwa north of Katada, having a period of about 70 min., and the rectangular water basin between Imazu, Siraisi-jima, Take-jima, Hikone, Nagahama, and Kaizu. The Kôbe Marine Observatory[1] and one[2] of the writers (Nomitsu) have reported that the latter basin should have a period of 30 min. for the longitudinal oscillation and 22 min. for the transversal. The remaining region conceivable as another oscillating area is only the inlet having the mouth at B_1 or B_2 line shown in Fig. 1. In order to find the corresponding period, the mean depth of the bay is calculated by measuring the area between the contour lines, and the mouth correction is obtained by exterpolating the usual formula established when the ratio of breadth to length is less than unity. The result is given as follows.

Table 6.

Calculated periods of the Imazu Bay.

Mouth line	Length	Breadth	Coefficient of mouth correction	Mean depth	Period
B_1	1600 m	5600 m	1.37	19.8 m	10.5 min
B_2	1050	3500	1.37	11.8	8.9
			(Transversal oscillation)		10.9

Thus the seiches of the period of 8–9 min. which often actually occur at Imazu are the oscillations of this bay.

1. loc. cit.
2. These Memoirs, A, **18**, 227 (1935).

Second, for *the neighbourhood of Ômizo*, we calculated the periods assuming the region as a shelf on the one hand and as the bay on the other. In Fig. 5, O_1 is the mean of the cross-sections e, f, g, h shown in Fig. 4, and represents the shelf of Ômizo proper. O_2 is the mean of the cross sections a, b, c, d thus representing the shelf on the north of O_1 at about Funaki. The calculated periods corresponding to these shelves are given below.

Table 7

Calculated periods for the shelf near Ômizo.

T_1	T_2
14.4 min	7.9 min
15.0	8.8

From this we see that the 15 min. seiches continually occurring at Ômizo are the proper oscillations of the well-developed lake shelf O_1 there. It may be here noticed that when this kind of seiche occurs at Ômizo, the resembling seiche occurs on the opposite shore (Yanagawa and Dezaike). Comparing the simultaneous records, however, we note that the phases are not always the reverse of each other, and that the periods at the opposite side are always a little longer than at Ômizo, attaining 16–18 min. (cf. Fig. 3). Now, calculating the period of transversal oscillation of the middle part of Lake Biwa, we get 18.7 min. and the report of the Imperial Marine Observatory[1] gives 16.7 min. for the same oscillation. At any rate, this is the same as the faint oscillation of 18 min. period which occasionally occurs at Ômizo. Since this is comparatively near the period of the shelf-seiches at Ômizo, it seems that the former is the exciter of the latter. The lake is far deeper at the Ômizo-side than at the Yanagawa-side. Hence, if the seiche oscillates between them, the amplitude at Yanagawa and Dezaike should be far greater than at Ômizo, but the actual seiches of ca. 15 min. period at Ômizo are always a few times larger and more frequent and of longer duration than at the others.

Next if we calculate the period for the Ômizo inlet as a bay, we get the following value, which explains the actual seiche of 12 min.

Table 8.

Calculated period of Ômizo bay.

Length	Breadth	Coefficient of mouth correction	Mean depth	Period
1300 m	3400 m	1.36	7.90 m	13.3 min

1. loc. cit.

The occasionally found period of 8–9 min. is either the oscillation which is propagated from the aforementioned oscillation of the O_2-shelf, or the harmonics of the transversal oscillation of 17–19 min. between Ômizo and the opposite shore.

Last, for *the neighbourhood of Kaizu*, we did not take special sounding ourselves. But utilising the data in the report of the Kôbe Marine Observatory we get the K-curve in Fig. 5 (B) as the mean cross section of the region. Here also we certainly perceive a shelf, though it is not so good in development and is rather narrow in breadth, so that oscillations of remarkable amplitude may not be expected. From the view-point as a bay, however, the length is longer than the breadth of the shelf, and therefore the bay-oscillation can have a longer period and a larger amplitude according to circumstances. At any rate, we calculated the periods of both the bay- and the shelf-oscillations as before, and obtained the following results.

Table 9

Calculated period for Kaizu region.

Oscillating region	Length	Breadth	Mean depth	Period
Bay	1300 m	3100 m	10.8 m	11.1 min
Shelf	1000		7.12 (Integral mean used)	8.0 (8.7)

The table shows that the seiche of 11–12 min. often observed at Kaizu is perhaps the bay-oscillation and that the 9 min. period is that of the shelf-oscillation. The report[1] of the Kôbe Marine Observatory gives an oscillating area (the *enclosed*, not the bay-type) of a 10.5 min. period in front of Kaizu and Imazu, but we can not understand it. It will be, indeed, altogether absurd to suppose that an oscillating region of the *enclosed* type has one of its sides in the deepest portion (about 90 m.) of the lake.

Finally the seiche of 30 min. period occurring most frequently at Kaizu must be the same as the oscillation of the identical period at Imazu.

V. Conclusion

Summarizing the chief points stated above, we see that:

1) The oscillations of the lake-shelves certainly exist, and that they have comparatively short periods among the lake-seiches.

1. loc. cit. Fig. 43, Map of Oscillating Areas.

In the ocean, as we have seen previously,[1] rather long periods among the secondary undulations belong to the shelf-seas, because there are no oscillations of the whole ocean except the tide, and because the shelf-seas are usually larger than the neighbouring bays. On the contrary, in a lake, any shelf is narrower than the whole breadth of the lake, so the shelf-oscillations commonly have comparatively short periods among the seiches.

2) Even in the bay of which the breadth is larger than the length, oscillations seem to occur beyond our expectation; especially so when the bay is fan-shaped. The amplitudes, however, are naturally small in any case.

We wish to express here our hearty thanks to the Imperial Academy for the grant with which we could accomplish the present research. Thanks are also due to Y. Toyohara, H. Nishida and M. Ôi for their assistance in making the soundings and observations.

1. These Memoirs, A, **18**, 213 (1935).

Combined Mirror and Lever Extensometer

By

Miyabi Sugihara

(Received Jan. 20, 1937)

Abstract

This instrument is a combined form of Martens' lever extensometer[1] and his mirror one.[2] It consists of two clips carrying two small rotating mirrors, being held on opposite sides of the specimen by a spring. Two double knife edges to which the levers are attached, are pivoted in shallow seats formed in the clips. The lower ends of the levers rotate the mirrors whose angles of rotation are measured by the telescopes and the scales. The two mirrors rotate in opposite directions when the specimen is elongated by applying a certain load. When adapted to determine the least stress at which Hooke's Law ceases to be exact the instrument attains a great magnification. The magnification obtained with the present instrument is about 7000.

Arrangement

This instrument, shown in Fig. 1, is so arranged that it can easily measure mechanically a small elongation of a solid rod. The two clips (CC) are held on opposite sides of the test-rod (R) by a spring (S) which rests in grooves (GG). Each clip is pointed at the lower end which is gripped directly on the test-rod, while the upper end has the double knife edge (N) interposed between it and the rod. Each mirror (M) is mounted on a frame which is fixed to each clip by means of the pivots (HH) centred in small holes drilled in the glass. The frame has an adjusting screw (K) which balances the mirror on the side opposite the clip. An arm (A) which is firmly fixed to the centre of each double knife edge, has a lever (L) clamped perpendicularly to the arm at one end, while the other end carries an adjusting screw (F) which balances the lever. A pivot (J) attached to the back side of each mirror is parrallel to the axis of rotation of the mirror, and is loosely inserted in the U-shaped end (U) of each lever.

Fig. 2a is a schematic sketch of a clip combined with a lever. The instrument is arranged in the following way: The upper part (AB) of the lever is perpendicular to the line IA joining both ends of the double knife edge. The central line along the part (AB) of

1) Batson and Hyde, Mechanical Testing, Vol. I, § 114.
2) „ „ „ § 115.

Fig. 1

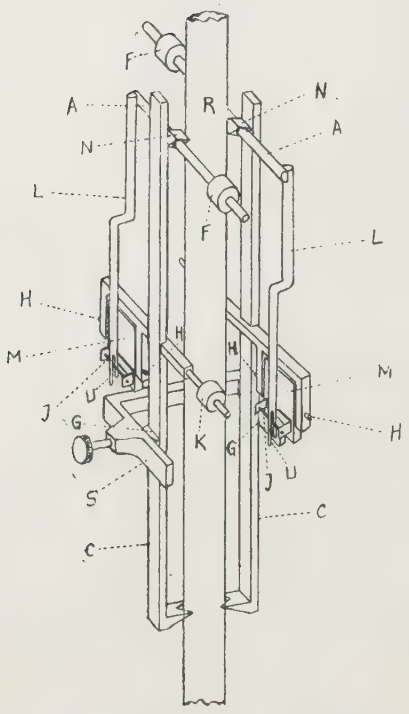
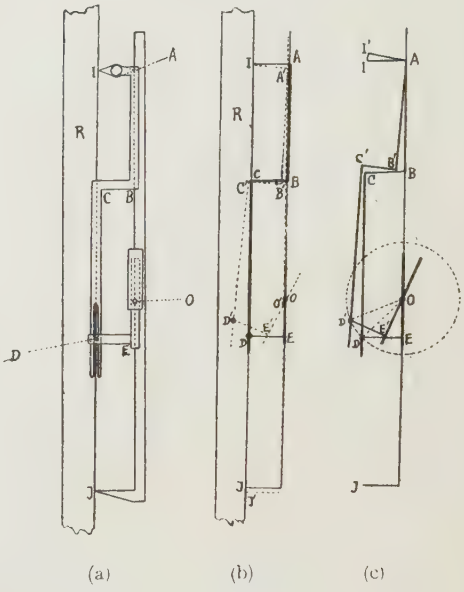


Fig. 2



the lever and the rotating axis (O) of the mirror lie in the same plane as the inner side of the clip. The middle part (BC) of the lever is perpendicular to both the upper part (AB) and the lower part (CD) of the lever; accordingly AB is parallel to CD. The main parts of the instrument have approximately the following sizes.

The double knife edge: IA = 5 mm.

The upper part of the lever: AB = 15 mm.

The middle part of the lever: BC = 5 mm.

The lower part of the lever: CD = 35 mm.

The gauge length: IJ = 100 mm.

The distance between the axis of rotation (O) of the mirror and the shallow seat formed in the clip to which the double knife edge is set: AO = 45 mm.

The distance from the pivot (D) inserted in the U-shaped end of the lever to the plane OE parallel to the mirror surface: DE = 5 mm.

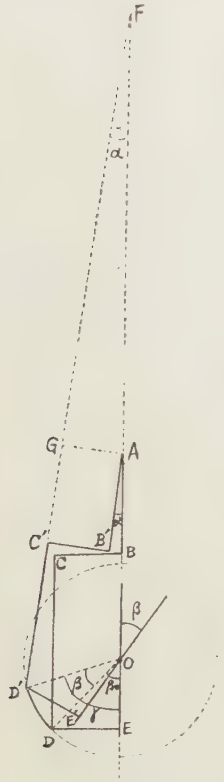
The distance between the axis of rotation (O) of the mirror and the projection E of the pivot to the plane (OE): OE = 5 mm.

Mechanism

At first the position of the mirror is adjusted in such a way as to be parallel to the inner surface of the clip as is shown in Fig. 2a. The motion of the test-rod under load is transmitted through the double knife edge to the lever which rotates the mirror. The angle of rotation of the mirror is measured by a telescope and a scale as in Martens' mirror extensometer.

In Fig. 2b the lines IA, ABCD, OE, and AOJ show the initial positions of the double knife edge, the lever, the mirror and the clip respectively, and the dotted lines IA′, A′B′C′D′, O′E′ and A′O′J′ show the final positions corresponding to the above-mentioned parts when the test-rod is loaded. The axis of rotation (O) of the mirror is considered to move with almost the same amount as the contact point of the double knife edge with the clip, that is $AA'=OO'$. In order to obtain the angle of rotation of the mirror, we may consider that the points J, O and A are fixed in their positions, and the lever is rotated around the point A by elongating the test-rod, as is shown in Fig. 2c. Thus the displacement AA′ of one end of the double knife edge as shown in Fig. 2b is now represented by the displacement I′I of the other end of the double knife edge in Fig. 2c. By the rotation of the mirror the pivot D fixed on the back side of the mirror moves to a point D′ on the circle described with the radius OD around the fixed centre O. Fig. 3 is drawn, for the sake of clearness, in calculating the amount of elongation of the test-rod under load.

Fig. 3



In the following, the friction at such contact points as the rotating axis of the mirror, the pivot on the back side of the mirror, and both the ends of the double knife edge, is neglected as a minor factor.

If $DE=b$, $OE=c$, $OD=e$ in Figs. 2c and 3, then $e=\sqrt{b^2+c^2}$. When the double knife edge rotates in a small angle α for the elongation δl of the test-rod, the lever also rotates in the same angle in Fig. 2c, so that $\angle IAI'=\angle BAB'=\alpha$. If $IA=a$, we have approximately $\delta l=a\alpha$. In Fig. 3

let β be the angle of rotation of the mirror which corresponds to the angle of rotation α of the lever, then $\angle DOD'=\beta$. Denote $\angle DOE$ by β_0, and $\angle D'OE$ by γ, then $\gamma=\beta_0+\beta$. Produce BA and D'C' so as to meet at a point F, then $\angle AFD'=\alpha$. Draw AG perpendicular to FD' from A and denote AG by p, then $BC=B'C'=AG=p$.

From ΔFAG we get $AF=p\,\mathrm{cosec}\,\alpha$.

From $\Delta FD'O$ we have $\dfrac{\sin\alpha}{OD'}=\dfrac{\sin(\gamma-\alpha)}{OF}$,

that is $\dfrac{\sin\alpha}{c}=\dfrac{\sin(\gamma-\alpha)}{p\,\mathrm{cosec}\,\alpha+f}$,

where AO is denoted by f.

Therefore $\sin\alpha=\dfrac{c}{f}\sin(\gamma-\alpha)-\dfrac{p}{f}$.

For small value of α, as is actually the case, the above equation becomes $\alpha=\dfrac{c}{f}\sin(\gamma-\alpha)-\dfrac{p}{f}$.

In the present case $b=5$ mm., $c=5$ mm., $p=5$ mm., and $f=45$ mm., accordingly $c=1/50$, $\beta_0=45^\circ$, and $\gamma-\alpha=45^\circ+(\beta-\alpha)$.

Therefore $\alpha=\dfrac{c}{f}\dfrac{1}{\sqrt{2}}\left\{\cos(\beta-\alpha)+\sin(\beta-\alpha)\right\}-\dfrac{p}{f}$.

But $\dfrac{c}{\sqrt{2}}=5$, so that $\dfrac{c}{\sqrt{2}}=p$.

Therefore $\alpha=\dfrac{p}{f}\left\{\cos(\beta-\alpha)+\sin(\beta-\alpha)-1\right\}$.

As $\beta-\alpha$ is small in the experiment with this instrument, by expanding the sine and the cosine of $\beta-\alpha$ in the above equation in its power series, we have

$$\begin{aligned}\alpha &= \frac{p}{f}\left\{\left[1-\frac{(\beta-\alpha)^2}{\lfloor 2}+\frac{(\beta-\alpha)^4}{\lfloor 4}-\cdots\cdots\right]\right.\\ &\quad\left.+\left[(\beta-\alpha)-\frac{(\beta-\alpha)^3}{\lfloor 3}+\frac{(\beta-\alpha)^5}{\lfloor 5}-\cdots\cdots\right]-1\right\}\\ &= \frac{p}{f}(\beta-\alpha)\left\{1-\frac{\beta-\alpha}{\lfloor 2}-\frac{(\beta-\alpha)^2}{\lfloor 3}\right.\\ &\quad\left.+\frac{(\beta-\alpha)^3}{\lfloor 4}+\frac{(\beta-\alpha)^4}{\lfloor 5}-\cdots\cdots\right\}.\end{aligned}$$

If we neglect the terms of the order higher than $\beta-\alpha$ as first approximation, we have

$$\alpha=\frac{p}{f}(\beta-\alpha),$$

that is $$\alpha=\frac{p}{f+p}\beta.$$

Substituting $p=5$, and $f=45$, we get

$$\alpha=\frac{1}{10}\beta.$$

Accordingly $$\beta-\alpha=\frac{9}{10}\beta.$$

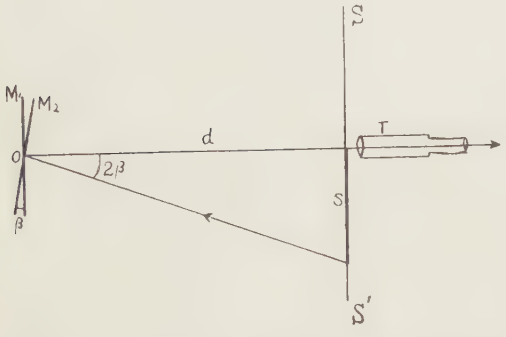
If we neglect the terms of the order higher than $(\beta-\alpha)^2$, and then by substituting $\beta-\alpha=\frac{9}{10}\beta$ for $\beta-\alpha$ on the right hand side of the above equation, we have as the second approximation,

$$\alpha=\frac{1}{10}\beta\left(1-\frac{81}{200}\beta\right).$$

Further neglecting the terms of the order higher than $(\beta-\alpha)^3$, and substituting the above value of α obtained as the second approximation, we have, by neglecting the terms of the order higher than β^4, as the third approximation for the terms of the second and the third power of $\beta-\alpha$

$$\alpha=\frac{1}{10}\beta\left(1-\frac{81}{200}\beta-\frac{3159}{20000}\beta^2\right).$$

Fig. 4



The angle of rotation of the mirror is observed by a telescope and a scale apart from the mirror. This arrangement is shown diagrammatically in Fig. 4. In this figure M_1 represents the initial position of the mirror, M_2 its final position, T the telescope and SS′ a scale whose distance from the mirror is denoted by d. If s denotes the scale reading corresponding to the angle of rotation β of the mirror, then

$$s=d\tan 2\beta.$$

Now let us consider the case of a very large elongation, and if we take $d=1650$ mm., and $s=330$ mm., then

$$\tan 2\beta=0.2.$$

Therefore $\beta=0.1$ radian,

which is small, and in such a case we have

$$s = 2d\beta.$$

In the case of such a small value of β as that mentioned above, the straight scale may be used. Because the amount to be corrected is negligibly small. In the case of a relatively large value of β, however, it is preferable to use a circular scale whose centre of curvature lies at the mirror. Because with such a scale the value of β is obtained directly from the scale reading without any correction.

Substituting the above value 0.1 of β in the correction terms $\frac{81}{200}\beta$ and $\frac{3159}{20000}\beta^2$ in the equation obtained as the third approximation, the correction terms take the values 0.04050 and 0.00158 respectively. Even for such a large value of β, the correction terms take such small values. As the value of β is smaller than the above in practice, the values of the correction terms become smaller, so that we may take only the first term. Hence we get as the relation between α and β the following equation,

$$\alpha = \frac{1}{10}\beta\left(1 - \frac{81}{200}\beta\right).$$

Similarly for the mirror which is mounted on the opposite clip, we get a similar relation such as

$$\alpha = -\frac{1}{10}\beta\left(1 + \frac{81}{200}\beta\right).$$

As is shown in Fig. 1, two clips are held on opposite sides of the specimen by a spring, and the two mirrors rotate in opposite directions as the specimen is elongated. Thus if we take the mean value of the two scale readings reflected from the mirrors, the difference of two scale readings resulting from a slight bending of the specimen and the correction terms in the above two equations cancel. Hence the proportionality between α and β is obtained.

Calculation of the constant

If we denote the mean values of each pair of α, β and s relating to two knife edges, two mirrors and two scales by $\bar{\alpha}$, $\bar{\beta}$ and $\bar{s}$ respectively, we have

$$\delta l = a\bar{\alpha},\ \bar{\alpha} = \frac{1}{10}\bar{\beta},\ \text{and}\ \bar{\beta} = \frac{\bar{s}}{2d}.$$

Therefore $\delta l = \frac{1}{20}\frac{a}{d}\bar{s}$.

Further, if we denote the gauge length IJ by l in Fig. 2, the elongation per unit length of the specimen is represented by

$$\frac{\delta l}{l} = \frac{1}{20} \frac{a}{ld} \bar{s} .$$

If we take $a = 5$ mm., $l = 100$ mm., $d = 1650$ mm., as is actually the case, then $\frac{\delta l}{l} = 0.0000015 15 \bar{s}$

where $\bar{s}$ is to be measured in mm. unit.

Testing

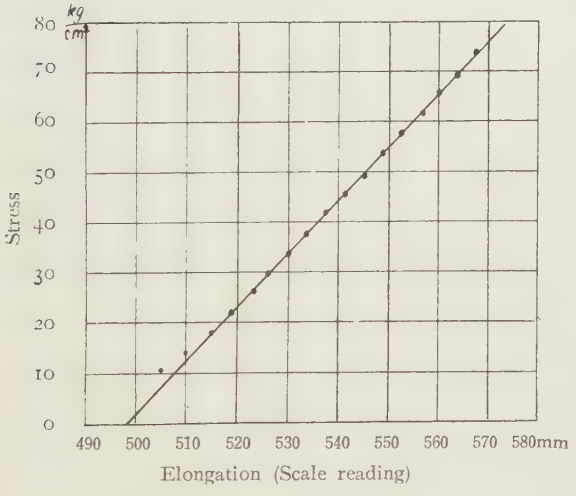
Within the elastic limit of a specimen the stress per unit area P is linearly proportional to the elongation per unit length $\frac{\delta l}{l}$ according to Hooke's Law, that is $P = E\frac{\delta l}{l}$

where E is the Young's modulus of the specimen. Hence

$$P = k\bar{s}$$

where k is a certain constant.

Fig. 5



It is shown by the above equation, that the stress per unit area and the scale reading are proportional. In order to test the above relation the writer employed a few aluminium rods of about 20 cm. in length and about 6 mm. in diameter. The upper end of each rod was clamped to a support and a load was applied to the lower end by a spring balance. He observed the scale readings corresponding to the gradually increased loads, and plotted a curve showing the relation between the stress per unit area and the scale reading. One such curve is shown in Fig. 5. From this curve it is clearly seen that the stress per unit area increases linearly with the scale reading except at the start. The deviation from the linear relation at a few points at the start, is of course due to a loose contact somewhere in the instrument.

Determination of the constant

The constant 0.000001515 as previously mentioned is obtained by assuming the following conditions: (1) The friction at the contact points in several parts is negligible, (2) Rough values of the sizes of various parts of the instrument are granted, (3) The plane containing the rotating axis of the mirror and the central axis of the upper part of the lever is supposed to coincide completely with the inner side of the clip. Hence we must admit that the above-mentioned constant may deviate more or less from the true constant of the instrument. Thus the writer experimentally determined the true constant by the following two methods.

Method I: He attached the present instrument and Martens' mirror extensometer of known constant to the same test-rod, and determined the constant for various elongations of the rod by comparing the two scale readings obtained with the two instruments.

Method II: He determined the constant by calculation from the stress-scale reading curve given in Fig. 5 which is obtained with a cold-worked aluminium rod, by adopting the average value of Young's modulus of aluminium set forth in his former research, "Young's Modulus of Aluminium Rod Composed of Large Crystal Grains."[1] The results obtained by the above two methods are almost the same, and the value 0.000001448 was obtained as the required constant.

With this value for the constant of the present instrument its magnification is 6906, when the telescope is 165 cm. away from the mirror, a relatively high magnification.

In conclusion the writer wishes to express his sincere thanks to Professor U. Yoshida for his kind advice and invaluable suggestions in constructing this instrument.

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1) These Memoirs, A, **17**, 389, (1934)

Limit of the Elasticity of an Aluminium Rod Composed of Comparatively Small Crystal Grains

By Miyabi Sugihara

(Received Jan. 20, 1937)

Abstract

When cold-worked commercial aluminium rods are treated by heat for a constant time at several temperatures below aluminium's melting point, the crystal grains grow larger with the increase of the temperature. The growth of the grains is not remarkable below about 200°C; but after exceeding that temperature, the growth becomes considerable. The limit of elasticity decreases slowly at a lower temperature, then rapidly and again slowly with the increase of annealing temperature. The relation between the limit of elasticity and the grain number per unit length is expressed very simply by an empirical formula.

Experimental Method

Fifteen commercial cold-worked aluminium rods about 20 cm. in length, and about 3 mm. in diameter were placed together in an electric furnace, and heated at a given temperature below the melting point for 2 days. After cooling to the room temperature, they were taken out of the furnace. The diameter of each rod was measured at several points by a micrometer screw gauge and the average value was taken, from which the cross-sectional area was calculated. Nine different groups of rods were subjected separately to different temperatures below the melting point, and afterwards each rod was tested for elongation.

In order to support the test-rod vertically, its upper end was fixed to a support and the lower end was clamped by a chuck of known weight. The upper end of a spring balance of known weight was hung by a hook fastened to the chuck and the lower end was clamped to a movable screw by which the spring balance might be pulled downwards. In this way the rod was loaded in various amounts which we intended. The extensometer[1] recently designed by the writer was attached to the rod whose elongation under the load was measured with two telescopes and scales which were 165 cm. apart from the mirrors.

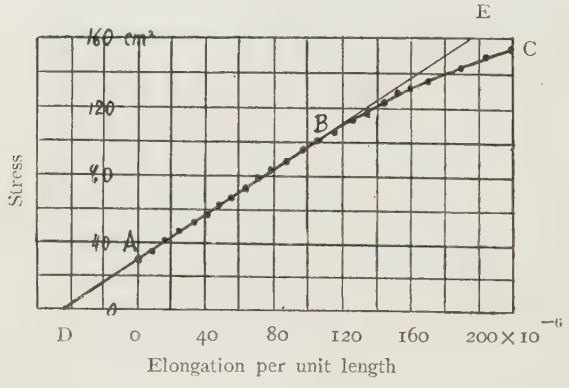
1) These Memoirs, A, **20**, 19, (1937).

In this way we took the mean value of the two scale readings made when the load was gradually increased.

The load applied to the rod was the sum of the readings of the spring balance, its weight, and the weights of the chuck, the rod and the extensometer. The weight of the rod and the extensometer might be neglected in comparison with the other weights owing to their smallness, the extensometer especially having been constructed as lightly as possible. The stress per unit area was easily calculated as the quotient of the total load divided by the average area of the rod.

We took, as the temporary zero point, the scale reading which was taken when the least possible load was initially applied to the rod. From the scale reading the elongation per unit length was calculated by using the constant[1] which had been determined for this extensometer.

Fig. 1



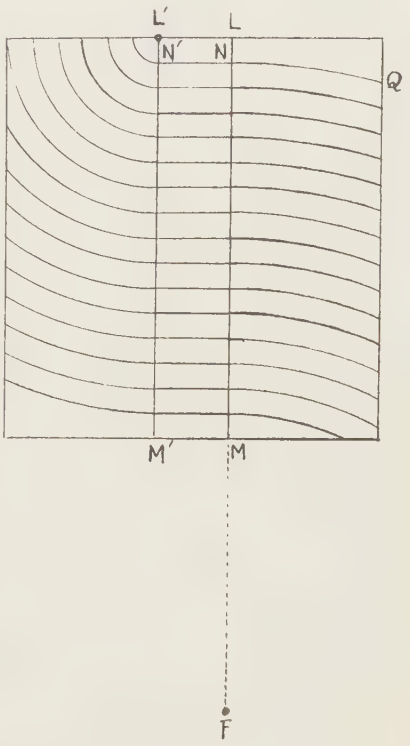
In order to show the relation between the stress per unit area and the elongation per unit length, the writer plotted the curve by taking the former as ordinate and the latter as abscissa. For example, one of the curves obtained is shown in Fig. 1. In this figure, if the curve ABC is produced to meet the abscissa at D, the point D will give a true zero point when no load was applied to the rod; and the elongation per unit length of the rod was obtained by taking D as a zero point. As is seen in Fig. 1, the part AB is a straight line on which Hooke's Law holds, and the curved part BC is beyond the elastic limit. If we draw a straight line ABE in such a way as to coincide with AB, and find the contact point of the line ABE with the curve BC, then this point gives the elastic limit required.

In order to determine such a contact point as accurately as possible, the writer employed an apparatus which may be called a "contact point finder". It is convenient in using this apparatus to consider the curve BC in Fig. 1 a circular arc in the vicinity of the elastic limit.

1) These Memoirs, A, **20**, 19, (1937).

As is shown in Fig. 2, two parallel lines LM and L′M′ are drawn on a thin transparent celluloid plate, and a number of parallel lines between them are drawn perpendicularly to the above two lines. One of them is denoted N′N. The line LM is prolonged to any point P, and an arc NQ is described with a radius PN with the point P as its centre. Similarly a number of arcs of different radii are described with P as the centre, by making them coincide with the straight lines between LM and L′M′ on the line LM. In the same way, another point is taken on the line L′M′ or on its prolongation, and many arcs are drawn with this point as the centre. For instance the point L′ is taken as such a point, and a number of arcs are described with L′ as the centre, so that these arcs meet the lines between LM and L′M′ on the line L′M′.

Fig. 2



Put the above-mentioned celluloid plate on the stress-strain curve as is shown in Fig. 1 in such a way that one of the lines between LM and L′M′, for example N′N, coincides with the line AB, and then displace the plate in the direction parallel to the line N′N and AB, and see whether the arc NQ coincides with the curve BC or not. When the coincidence is imperfect, apply another arc to BC. Thus if we find out a position of perfect coincidence on the curve, we are able to determine the point where the straight line ABE contacts with the curve BC, that is, the limit of elasticity. The result obtained by this method is, of course, slightly ambiguous, but it is convenient to determine easily and simply the limit of elasticity.

The writer used 8 to 12 aluminium rods for each group annealed at a certain temperature, and determined the limit of elasticity for each rod from the stress-strain curve by the above method, and then took the average value. The values of Young's modulus were calculated

from the curve as shown in Fig. 1, within the limit of elasticity, by dividing the stress per unit area by the elongation per unit length. One of the results is shown in Table I.

Experimental Result

The result shown in Table I was that of the specimens which were annealed at 130°C for 2 days. The table shows that the elastic limits are almost the same within the experimental error, and that the values of Young's modulus are also the same.

Table I

Limit of elasticity		Young's modulus
Stress per unit area	Elongation per unit length	in C. G. S unit
540 $\frac{kg}{cm^2}$	7.90×10^{-4}	6.70×10^{11}
530	7.80 „	6.66 „
520	7.65 „	6.66 „
510	7.52 „	6.63 „
565	8.22 „	6.73 „
565	8.30 „	6.66 „
540	7.95 „	6.66 „
530	7.73 „	6.72 „
575	8.33 „	6.76 „
535	7.90 „	6.64 „
Average 541	7.93×10^{-4}	6.68×10^{11}

The results obtained with various annealing temperatures are given in Table II. The annealing temperatures are shown in the first column in the table. The temperature 25°C in this column is the room temperature at which the elastic limits of cold-worked aluminium rods were measured. As the grain number per centimetre of the specimen at this temperature is very great, it is assumed to be infinitely great and is denoted by the sign ∞ in the last column in

Table II

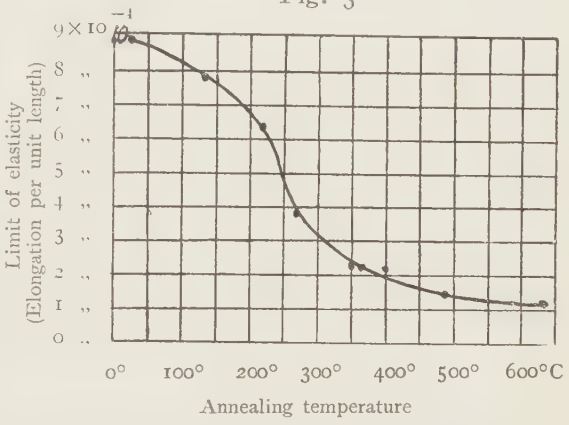
Annealing temperature	Limit of elasticity		Young's modulus	Grain number per unit length
in centigrade	Stress per unit area	Elongation per unit length	in C. G. S unit	per one cm.
25	603 $\frac{kg}{cm^2}$	8.88×10^{-4}	6.65×10^{11}	∞
130	541	7.93 „	6.68 „	21220
220	439	6.44 „	6.68 „	9084
270	264	3.87 „	6.67 „	3200
350	150	2.21 „	6.67 „	1515
370	150	2.20 „	6.70 „	1490
400	148	2.17 „	6.70 „	1303
480	98	1.44 „	6.68 „	1049
630	81	1.19 „	6.68 „	773

the table. The grain number was measured by examining the etched surface of the specimen through a microscope.

Table II shows that according to the increase of the annealing temperature the elastic limit as well as the grain number of each specimen decreases gradually, but that Young's modulus has almost the same value independent of crystal grain sizes. The writer has already reported that Young's modulus of aluminium rod composed of large crystal grains had almost the same value independent of the grain sizes[1]. The writer confirmed this point in the present research, the values of Young's modulus being almost the same in both of his experiments.

The relation between the annealing temperature and the limit of elasticity is shown in Fig. 3 by taking the former as abscissa and the latter as ordinate. The curve in this figure shows that with the increase of annealing temperature the limit of elasticity decreases rather slowly at first from about 25°C to about 200°C, then rapidly from about 200°C to about 300°C, and again slowly from about 300°C to about 630°C.

Fig. 3



In order to show the relation between the grain number per unit length and the limit of elasticity, a curve was plotted by taking the former as abscissa and the latter as ordinate. From the curve the writer approximately obtained an empirical formula as follows:

$$L = L_0(1 - e^{-\alpha z}),$$

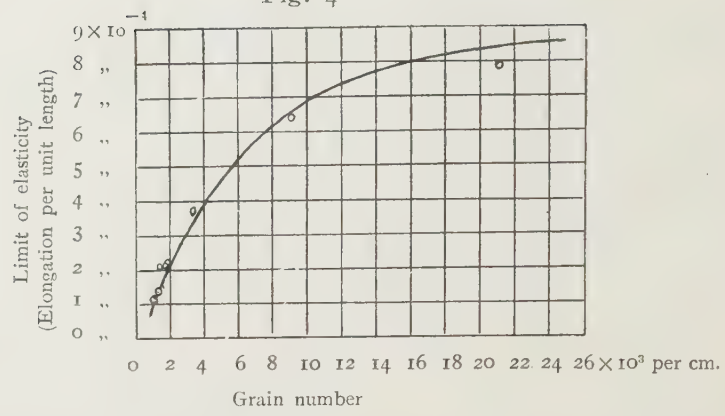
where z denotes the grain number per unit length, L the limit of elasticity represented by elongation per unit length, L_0 the value of L when z becomes infinity, e the base of natural logarithm and α a certain constant. For the constants α and L_0, the following values are found to accord with the results of the present experiment.

$$\alpha = 0.000151, \qquad L_0 = 8.88 \times 10^{-4}.$$

The curve in Fig. 4 is drawn with the above values of α and L_0

1) These Memoirs, A, **17**, 389, (1934).

Fig. 4



for various values of z, and the small circles for the observed values. The agreement between the calculation and the observation is almost perfect, except at the point of the largest grain number, where the exact counting of the grain number is very difficult.

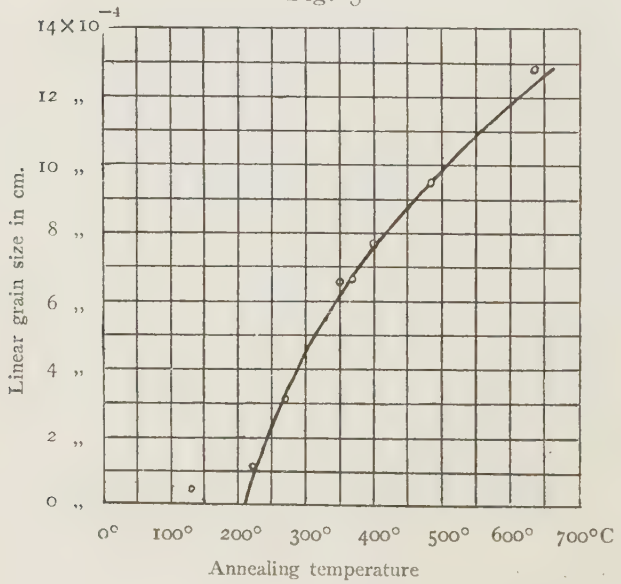
So far as the present research is concerned, the limit of elasticity increases with increasing grain number rather rapidly at first, and then slowly asymptotically approaches a definite saturation value of 8.88×10^{-4}.

Further, the writer plotted a curve which shows the relation between the annealing temperature and the linear grain size, that is, the reciprocal of the grain number per unit length, by taking the former as abscissa and the latter as ordinate. From the curve the writer obtained an empirical formula as follows:

$$Y = A(T - 200)^{\frac{1}{2}} + B,$$

where Y denotes

Fig. 5



the linear grain size, T the annealing temperature, and A and B are constants which take 0.73 and -2.7×10^{-4} respectively.

The curve drawn in Fig. 5 is that calculated from the above empirical formula, and the small circles denote the observed values. The coincidence seems to be satisfactory in this kind of experiment. From the curve it is clear that the crystal grains almost cease to grow at the temperature below about 200°C, but after exceeding 200°C they grow rapidly. This indicates that the recrystallization temperature is higher than about 200°C in the case of the test pieces of aluminium examined in the present experiment. By calculating the grain size at the melting point 657°C of aluminium from the above formula, the value 12.86×10^{-4} cm. was obtained.

Note: The relation between the grain size and the annealing temperature already has been studied by Professor U. Yoshida and Mr. S. Nagata[1]. They preliminarily annealed cold-worked aluminium plate, then elongated it by a certain small amount and at last annealed it again at a higher temperature below its melting point. That they annealed the specimen in two stages makes their experiment different from the present one. The discrepancy between their research and the present one is due to a different heat treatment and the degree of cold working.

In conclusion the writer wishes to express his sincere thanks to Professor U. Yoshida for his kind guidance in this experiment.

1) These Memoirs, A, **19**, 169, (1936).

Changes in the Density of Silver by Cold Hammering

By Atuomi Igata

(Received Jan. 20, 1937)

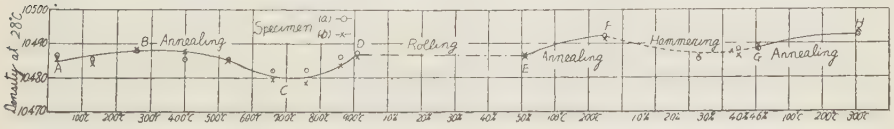
Abstract

The change of density of pure bar silver, caused by hammering applied during various stages of its heat treatment, was investigated. Herein is discussed on the basis of the experiments the measure in which small cavities in the metal can be vanished by hammering.

In this experiment silver, more than 99.95% pure, was used as a specimen. It was made into a long bar, the cross section of which was 4 mm × 10 mm, first by casting into a bar, next by rolling (to about 50% reduction in thickness), and then tempered for a few minutes at a temperature of about 500°C. The annealing of the specimen was made in a vacuum furnace. It was first raised to a definite temperature in about 30 or 40 minutes; was kept there for about 3 hours; then was left in the furnace to cool. The density of the specimen was measured exactly in the same manner as that described in the previous paper[1]. In the previous experiment four different states of a silver specimen were respectively named A, B, C and D, and their significance was studied. It was argued that if the specimen in the D state, after being subjected by cold rolling to about 60% reduction in thickness, was annealed at about 250°C, it would regain almost the standard density, free from internal stress and minute cavities. Let this process be named the D. R. A. (D state-Rolling-Annealing) process.

In the present experiment the change of the density caused by hammering with a screw presser or a steam hammer at various states of the specimen was studied. Rolling and annealing were also studied as auxiliary experiments for discussion purposes. First of all, two

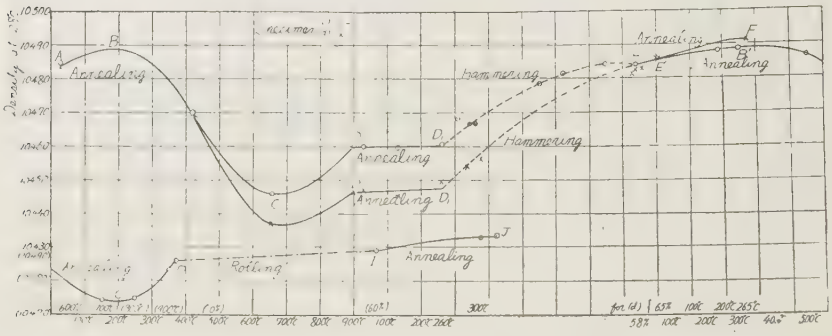
Fig. 1



1) These Memoirs, A, **19**, 215 (1936).

specimens (a) and (b) were brought to the standard state and standard density by means of the D. R. A. process, as is shown in Fig. 1, state F. Then they were reduced 46% in thickness by cold hammering. The decrease of density by this process of hammering was about $4/10^4$, but it was recovered entirely by annealing at 300°C. This fact was also observed in the parts B-E-B_1 and B-E′-B′ of the curves in Fig. 3, obtained with the specimens (e) and (f) respectively; and also in the part A-P-B of the curve in Fig. 4, which was obtained with the specimen (i).

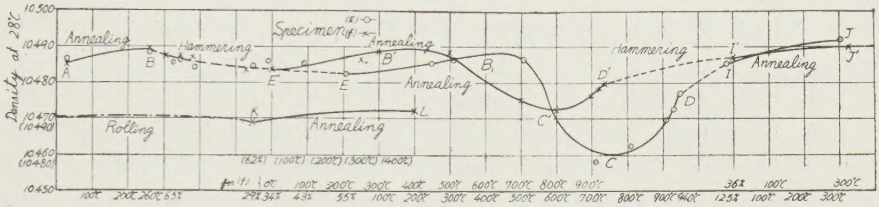
Fig. 2



Next by annealing the specimens (c) and (d) successively up to the temperature of 900°C, they were brought to the D state as is seen in Fig. 2. By a subsequent annealing at a temperature up to about 260°C, they were brought to the D_1 state, which was exactly the same as the D state. In the D or D_1 states the specimens still have some minute cavities corresponding to the incomplete recovery of their density. Next when reduced by hammering to about 60% their former thickness the densities increased almost to their value in the A state; and then the reduction in their densities caused by the process of hammering was recovered nearly by annealing at 270°C. From this fact it may be inferred that the minute cavities in the metal are nearly destroyed by hammering. But judging from the fact that the value of the density at the point B′ of the specimen (c) in Fig. 2 (also at the point J′ of the specimen (f) in Fig. 3) is somewhat lower than the standard value, it may be supposed that minute cavities can hardly be vanished completely by cold hammering. However, the question still remains as to whether the minute cavities are a real remainder or something newly made in the process of hammering. If the above supposition is true, then the density of the specimen (c) ought to be brought to the standard

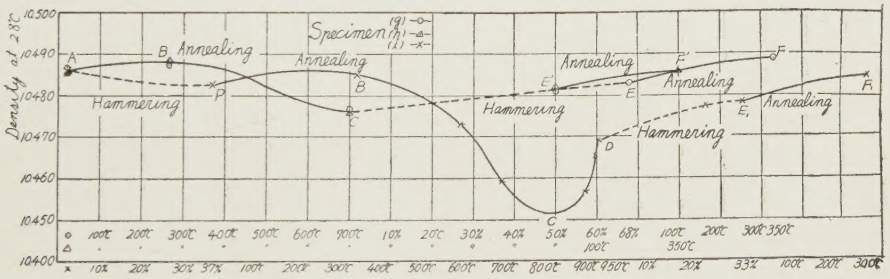
value by the D. R. A. process. This point was confirmed, as is seen by the lower curve B′-C′-D′-I-J′ in Fig. 2 (and J′-K-L for another specimen (f) in Fig. 3).

Fig. 3



In Fig. 3 the specimens (e) and (f) were first brought to the B state, then after several successive hammerings producing 55% and 34% reduction in thickness respectively, the annealing process was applied; but they underwent the same change as that displayed previously in Fig. 2.

Fig. 4



In Fig. 4 the writer hammered and annealed specimens (g) and (h) starting from the state C, and found that the disappearance of minute cavities was incomplete, as is revealed by the lower values of their densities at the points F and F′. The same thing was also true of specimen (i) which, starting from the state D, was hammered to 38% and annealed at 310°C, bringing it to the F_1 state where the density was much lower than the standard value. Considering these facts, it seems almost certain that the vanishing of minute cavities by hammering is not complete.

From these experiments the writer has been led to the following conclusion. When any specimen of pure silver is hammered, the 30%–50% reduction in thickness is accompanied by the diminution of about $4/10^4$ in its density, no matter what the state and history of the specimen may be, exclusive of the destruction of the small cavities by hammering. For recovery from the diminution of the density caused by hammering of such a degree, annealing at about 300°C—400°C is

required. The minute cavities in the metal are nearly all but not entirely destroyed by hammering, as in the case of rolling.

Lastly, the writer studied the change of density of pure silver, which was caused by stretching ; and found that this change was also a diminution which was greater than that caused by hammering or by rolling.

In conclusion, the writer wishes to express his hearty thanks to Prof. U. Yoshida of the Kyoto Imperial University for his kind guidance throughout this research.

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